AutoMoG Pyomo Model

June 1, 2021

Sets, Parameters and Variables

Table 1: Sets of the MIP.

Set	**Defined for**	**Meaning**
P	–	Products
T	–	Time steps
C	–	Components
U_c	$\forall c \in C$	Subcomponents
$Z_{c,u}$	$\forall c \in C, u \in U_c$	States
R_c	$\forall c \in C$	Revisions
S	–	Storages
$B_{c,u,z}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}$	Breakpoints
$X_{c,u}$	$\forall c \in C, u \in U_c$	Input Products of Subcomponents
$Y_{c,u}$	$\forall c \in C, u \in U_c$	Output Products of Subcomponents

Table 2: Parameters of the MIP.

Param.	**Set**	**Meaning**	**Domain**
τ_t	$\forall t \in T$	Time step	$\mathbb{Z}_0^+$
h_p^{source}	$\forall p \in P$	Product has source	{0,1}
h_p^{sink}	$\forall p \in P$	Product has source	{0,1}
$c_{p,t}^{\text{buy}}$	$\forall p \in P, t \in T$	Price of bought product	$\mathbb{R}_0^+$
$c_{p,t}^{\text{sell}}$	$\forall p \in P, t \in T$	Price of sold product (compensation)	$\mathbb{R}_0^+$

Continuation on the next page

Table 2: Parameters of the MIP.

Param.	**Set**	**Meaning**	**Domain**
$d_{p,t}$	$\forall p \in P, t \in T$	Demand of product	$\mathbb{R}_0^+$
$\omega_{c,u}^{\text{out}}$	$\forall c \in C, u \in U_c$	Product that is output of a component	P
$\omega_{c,u}^{\text{in}}$	$\forall c \in C, u \in U_c$	Product that is output of a component	$\mathbb{R}_0^+$
$r_{c,p}$	$\forall c \in C, p \in \bigcup_{u \in U_c} X_{c,u} \cup Y_{c,u}$	Ramping parameter	$\mathbb{R}_0^+$
$d_c^{\text{min,down}}$	$\forall c \in C$	Minimum downtime	$\mathbb{R}_0^+$
$d_c^{\text{min,up}}$	$\forall c \in C$	Minimum uptime	$\mathbb{R}_0^+$
c_c^{startup}	$\forall c \in C$	Specific startup costs	$\mathbb{R}_0^+$
$\rho_{c,r}^{\text{duration}}$	$\forall c \in C, r \in R_c$	Time per revision and unit	$\mathbb{R}_0^+$
ρ_c^{Δ}	$\forall c \in C$	Time between revisions	$\mathbb{R}_0^+$
w_s^{min}	$\forall s \in S$	Minimum fill level	$\mathbb{R}_0^+$
w_s^{max}	$\forall s \in S$	Maximum fill level	$\mathbb{R}_0^+$
$\dot{w}_s^{\text{max,load}}$	$\forall s \in S$	Maximum loading rate	$\mathbb{R}_0^+$
$\dot{w}_s^{\text{max,unload}}$	$\forall s \in S$	Maximum unloading rate	$\mathbb{R}_0^+$
η_s^{load}	$\forall s \in S$	Storage loading efficiency	$\mathbb{R}_0^+$
η_s^{unload}	$\forall s \in S$	Storage unloading efficiency	$\mathbb{R}_0^+$
p_s	$\forall s \in S$	Storage product	
$x_{c,u,z,b,t}^{\text{domain}}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}, b \in B_{c,u,z}, t \in T$	Domain breakpoint	$\mathbb{R}_0^+$
$x_{c,u,z,b,t}^{\text{range}}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}, b \in B_{c,u,z}, t \in T$	Range breakpoint	$\mathbb{R}_0^+$
$\Phi_{c,p}^{\text{in,init}}$	$\forall c \in C, p \in \bigcup_{u \in U_c} X_{c,u}$	Initial product input of component	$\mathbb{R}_0^+$
$\Phi_{c,p}^{\text{out,init}}$	$\forall c \in C, p \in \bigcup_{u \in U_c} Y_{c,u}$	Initial product output of component	$\mathbb{R}_0^+$
δ_c^{init}	$\forall c \in C$	Indicates if component is initially active	$\{0,1\}$
$\delta_{c,u,z}^{\text{state,init}}$	$\forall c \in C$	Indicates if component state is initially active	$\{0,1\}$
$d_c^{\text{up,init}}$	$\forall c \in C$	Initial uptime of component	$\mathbb{R}_0^+$
$d_c^{\text{down,init}}$	$\forall c \in C$	Initial downtime of component	$\mathbb{R}_0^+$
W_s^{init}	$\forall s \in S$	Initial fill level of storage	$\mathbb{R}_0^+$

Table 3: Variables of the MIP.

Var.	**Set**	**Meaning**	**Domain**
$\delta_{c,t}$	$\forall c \in C, t \in T$	Component and its subcomponents are active	$\{0,1\}$
$\delta^{\text{state}}_{c,u,z,t}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T$	Comp. and its subcomps. are active in specific state	$\{0,1\}$
$\Phi^{\text{out}}_{c,p,t}$	$\forall c \in C, p \in \bigcup_{u \in U_c} Y_{c,u}, t \in T$	Product output of component	$\mathbb{R}^+_0$
$\Phi^{\text{in}}_{c,p,t}$	$\forall c \in C, p \in \bigcup_{u \in U_c} X_{c,u}, t \in T$	Product input of component	$\mathbb{R}^+_0$
$\Phi^{\text{out,sub}}_{c,u,z,t}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T$	Product output of subcomponent	$\mathbb{R}^+_0$
$\Phi^{\text{in,sub}}_{c,u,z,t}$	$\forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T$	Product input of subcomponent	$\mathbb{R}^+_0$
$C^{\text{startup}}_{c,t}$	$\forall c \in C, t \in T$	Startup costs of component	$\mathbb{R}^+_0$
$\delta^{\text{startup}}_{c,t}$	$\forall c \in C, t \in T$	Indicates if component is starting up	$\{0,1\}$
$\delta^{\text{shutdown}}_{c,t}$	$\forall c \in C, t \in T$	Indicates if component is shutting down	$\{0,1\}$
$\delta^{\text{revision}}_{c,t}$	$\forall c \in C, t \in T$	Component is revised	$\{0,1\}$
$\delta^{\text{revision,indic}}_{c,r,t}$	$\forall c \in C, r \in R_c, t \in T$	Indicates if component is being revised	$\{0,1\}$
$\dot{W}^{\text{load}}_{s,t}$	$\forall s \in S, t \in T$	Loading rate of storage	$\mathbb{R}^+_0$
$\dot{W}^{\text{unload}}_{s,t}$	$\forall s \in S, t \in T$	Unloading rate of storage	$\mathbb{R}^+_0$
$W_{s,t}$	$\forall s \in S, t \in T$	Fill level of storage	$\mathbb{R}^+_0$
$\delta^{\text{load}}_{s,t}$	$\forall s \in S, t \in T$	Indicates if storage is loaded	$\{0,1\}$
$\delta^{\text{unload}}_{s,t}$	$\forall s \in S, t \in T$	Indicates if storage is unloaded	$\{0,1\}$
$O^{\text{sink}}_{p,t}$	$\forall p \in P, t \in T$	Sink used	$\mathbb{R}^+_0$
$O^{\text{source}}_{p,t}$	$\forall p \in P, t \in T$	Source used	$\mathbb{R}^+_0$
$\Delta D_{p,t}$	$\forall p \in P, t \in T$	Unfullfilled demand	$\mathbb{R}^+_0$

Objective Function

$$\min \sum_{t \in T} \left(\sum_{p \in P} \left(O^{\text{source}}_{p,t} c^{\text{buy}}_{p,t} - O^{\text{sink}}_{p,t} c^{\text{sell}}_{p,t} + 1000 \Delta D_{p,t} \right) + \sum_{c \in C} C^{\text{startup}}_{c,t} \right) \tag{1}$$

Constraints

Product Constraints

Product Balance

$$
\begin{aligned}
d_{p,t} + \Delta D_{p,t} = & \sum_{\substack{c \in C \\ p \in \bigcup_{u \in U_c} Y_{c,u}}} \Phi^{\text{out}}_{c,p,t} - \sum_{\substack{c \in C \\ p \in \bigcup_{u \in U_c} X_{c,u}}} \Phi^{\text{in}}_{c,p,t} + \sum_{s \in S} p_s \left(\eta^{\text{unload}}_s \dot{W}^{\text{unload}}_{s,t} - \dot{W}^{\text{load}}_{s,t} \right) \\
& + h^{\text{source}}_p O^{\text{source}}_{p,t} - h^{\text{sink}}_p O^{\text{sink}}_{p,t} \quad \forall t \in T, p \in P \qquad (2)
\end{aligned}
$$

Production Constraints

Couple activity of subcomponents

$$
\sum_{z \in Z_{c,u}} \delta^{\text{state}}_{c,u,z,t} = \delta_{c,t} \quad \forall c \in C, u \in U_c, t \in T \qquad (3)
$$

Couple output/input of component and their subcomponents

$$
\Phi^{\text{out}}_{c,p,t} = \sum_{z \in Z_{c,u}} \Phi^{\text{out,sub}}_{c,u,z,t} \qquad \forall c \in C, u \in U_c, p \in Y_{c,u}, t \in T \qquad (4)
$$

$$
\Phi^{\text{in}}_{c,p,t} = \sum_{z \in Z_{c,u}} \Phi^{\text{in,sub}}_{c,u,z,t} \qquad \forall c \in C, u \in U_c, p \in X_{c,u}, t \in T \qquad (5)
$$

$$
\Phi^{\text{out}}_{c,p,t} = \sum_{\substack{u \in U_c \\ p = \omega_{c,u}}} \sum_{z \in Z_{c,u}} \Phi^{\text{out,sub}}_{c,u,z,t} \qquad \forall c \in C, p \in P, t \in T \qquad (6)
$$

$$
\Phi^{\text{in}}_{c,p,t} = \sum_{\substack{u \in U_c \\ p = \omega_{c,u}}} \sum_{z \in Z_{c,u}} \Phi^{\text{in,sub}}_{c,u,z,t} \qquad \forall c \in C, p \in P, t \in T \qquad (7)
$$

Coupled component input and activation and output and activation

$$
\Phi^{\text{out,sub}}_{c,u,z,t} \leq \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{ x^{\text{domain}}_{c,u,z,b,t} \right\} \delta^{\text{state}}_{c,u,z,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \qquad (8)
$$

$$
\Phi^{\text{out,sub}}_{c,u,z,t} \geq \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{ x^{\text{domain}}_{c,u,z,b,t} \right\} \delta^{\text{state}}_{c,u,z,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \qquad (9)
$$

$$
\Phi^{\text{in,sub}}_{c,u,z,t} \leq \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{ x^{\text{range}}_{c,u,z,b,t} \right\} \delta^{\text{state}}_{c,u,z,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \qquad (10)
$$

$$
\Phi^{\text{in,sub}}_{c,u,z,t} \geq \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{ x^{\text{range}}_{c,u,z,b,t} \right\} \delta^{\text{state}}_{c,u,z,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \qquad (11)
$$

Piecewise linear coupling of component input and output (SOS2 Constraints)

$$\Phi^{\text{out,sub}}_{c,u,z,t} = \sum_{b \in B_{c,u,z}} x^{\text{domain}}_{c,u,z,b,t} \Phi^{\text{weight}}_{c,u,z,b,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \tag{12}$$

$$\Phi^{\text{in,sub}}_{c,u,z,t} = \sum_{b \in B_{c,u,z}} x^{\text{range}}_{c,u,z,b,t} \Phi^{\text{weight}}_{c,u,z,b,t} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \tag{13}$$

$$\sum_{b \in B_{c,u,z}} \Phi^{\text{weight}}_{c,u,z,b,t} = 1 \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, t \in T \tag{14}$$

$$\Phi^{\text{weight}}_{c,u,z,b,t} \in \text{SOS2} \quad \forall c \in C, u \in U_c, z \in Z_{c,u}, b \in B_{c,u,z}, t \in T \tag{15}$$

Maximum ramp-up and ramp-down rate

$$\begin{aligned} \Phi^{\text{in}}_{c,p,t} - \Phi^{\text{in}}_{c,p,t-1} \leq \quad & r_{c,p} \left(\delta_{c,t} + \delta^{\text{state}}_{c,u,z,t} - 1\right) \\ & + \max\left\{0, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,t}\right\} - r_{c,p}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,t} - \delta_{c,t-1}\right) \\ & + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,t}\right\} \cdot \left(1 - \delta^{\text{state}}_{c,u,z,t}\right) \\ & \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} X_{c,u}, t \in T \setminus \{0\} \end{aligned} \tag{16}$$

$$\begin{aligned} \Phi^{\text{in}}_{c,p,t-1} - \Phi^{\text{in}}_{c,p,t} \leq \quad & r_{c,p} \delta_{c,t} \\ & + \max\left\{r_{c,p}, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,t}\right\}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,t-1} - \delta_{c,t}\right) \\ & + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,t}\right\} \cdot \left(1 - \delta^{\text{state}}_{c,u,z,t-1}\right) \\ & \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} X_{c,u}, t \in T \setminus \{0\} \end{aligned} \tag{17}$$

$$
\begin{aligned}
\Phi^{\text{out}}_{c,p,t} - \Phi^{\text{out}}_{c,p,t-1} \leq \quad & r_{c,p}\left(\delta_{c,t} + \delta^{\text{state}}_{c,u,z,t} - 1\right) \\
& + \max\left\{0, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,t}\right\} - r_{c,p}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,t} - \delta_{c,t-1}\right) \\
& + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,t}\right\} \cdot \left(1 - \delta^{\text{state}}_{c,u,z,t}\right) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} Y_{c,u}, t \in T \setminus \{0\}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\Phi^{\text{out}}_{c,p,t-1} - \Phi^{\text{out}}_{c,p,t} \leq \quad & r_{c,p}\delta_{c,t} \\
& + \max\left\{r_{c,p}, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,t}\right\}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,t-1} - \delta_{c,t}\right) \\
& + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,t}\right\} \cdot \left(1 - \delta^{\text{state}}_{c,u,z,t-1}\right) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} Y_{c,u}, t \in T \setminus \{0\}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\Phi^{\text{in}}_{c,p,0} - \Phi^{\text{in,init}}_{c,p} \leq \quad & r_{c,p}\left(\delta_{c,0} + \delta^{\text{state}}_{c,u,z,0} - 1\right) \\
& + \max\left\{0, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,0}\right\} - r_{c,p}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,0} - \delta^{\text{init}}_{c}\right) \\
& + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,0}\right\} \cdot \left(1 - \delta^{\text{state}}_{c,u,z,0}\right) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} X_{c,u}
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\Phi^{\text{in,init}}_{c,p} - \Phi^{\text{in}}_{c,p,0} \leq \quad & r_{c,p}\delta_{c,0} \\
& + \max\left\{r_{c,p}, \min_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,0}\right\}\right\} \cdot \left(\delta^{\text{state,init}}_{c,u,z} - \delta_{c,0}\right) \\
& + \max_{\substack{b \in B_{c,u,z} \\ b \neq 0}} \left\{x^{\text{domain}}_{c,u,z,0}\right\} \cdot \left(1 - \delta^{\text{state,init}}_{c,u,z}\right) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u \in U_c} X_{c,u},
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\Phi^{\text{out}}_{c,p,0} - \Phi^{\text{out,init}}_{c,p} \leq \quad & r_{c,p}\left(\delta_{c,0} + \delta^{\text{state}}_{c,u,z,0} - 1\right) \\
& + \max\left\{0, \min_{\substack{b\in B_{c,u,z}\\ b\neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,0}\right\} - r_{c,p}\right\} \cdot \left(\delta^{\text{state}}_{c,u,z,0} - \delta^{\text{init}}_{c}\right) \\
& + \max_{\substack{b\in B_{c,u,z}\\ b\neq 0}} \{x^{\text{domain}}_{c,u,z,0}\} \cdot (1 - \delta^{\text{state}}_{c,u,z,0}) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u\in U_c} Y_{c,u} \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\Phi^{\text{out,init}}_{c,p} - \Phi^{\text{out}}_{c,p,0} \leq \quad & r_{c,p}\delta_{c,0} \\
& + \max\left\{r_{c,p}, \min_{\substack{b\in B_{c,u,z}\\ b\neq 0}} \left\{x^{\text{domain}}_{c,u,z,b,0}\right\}\right\} \cdot \left(\delta^{\text{state,init}}_{c,u,z} - \delta_{c,0}\right) \\
& + \max_{\substack{b\in B_{c,u,z}\\ b\neq 0}} \{x^{\text{domain}}_{c,u,z,0}\} \cdot (1 - \delta^{\text{state,init}}_{c,u,z}) \\
& \forall c \in C, u \in U_c, z \in Z_{c,u}, p \in \bigcup_{u\in U_c} Y_{c,u} \qquad (23)
\end{aligned}
$$

Startup/shutdown of unit

$$\delta_{c,t-1} - \delta_{c,t} - \delta^{\text{shutdown}}_{c,t} + \delta^{\text{startup}}_{c,t} = 0 \qquad \forall c \in C, t \in T | t > 0 \qquad (24)$$

$$\delta^{\text{init}}_{c} - \delta_{c,0} - \delta^{\text{shutdown}}_{c,0} + \delta^{\text{startup}}_{c,0} = 0 \qquad \forall c \in C \qquad (25)$$

$$\delta^{\text{shutdown}}_{c,t} + \delta^{\text{startup}}_{c,t} \leq 1 \qquad \forall c \in C, t \in T \qquad (26)$$

Startup cost of unit

$$C^{\text{startup}}_{c,t} = \delta^{\text{startup}}_{c,t} c^{\text{startup}}_{c} \quad \forall c \in C, t \in T \qquad (27)$$

Minimum up-time of unit

$$\sum_{\tau=t-d^{\text{min,up}}_{c}+1}^{t} \delta^{\text{startup}}_{c,\tau} \leq \delta_{c,t} \qquad \forall c \in C, t \in T | t - d^{\text{min,up}}_{c} + 1 \geq 0 \qquad (28)$$

$$\delta^{\text{shutdown}}_{c,t} = 0 \qquad \forall c \in C, t \in T | t + d^{\text{up,init}}_{c} - d^{\text{min,up}}_{c} + 1 \leq 0 \qquad (29)$$

Minimum down-time of unit

$$\sum_{\tau=t-d_c^{\text{min,down}}+1}^{t} \delta_{c,\tau}^{\text{shutdown}} \leq 1 - \delta_{c,t} \qquad \forall c \in C, t \in T | t - d_c^{\text{min,down}} + 1 \geq 0 \tag{30}$$

$$\delta_{c,t}^{\text{startup}} = 0 \qquad \forall c \in C, t \in T | t + d_c^{\text{down,init}} - d_c^{\text{min,down}} + 1 \leq 0 \tag{31}$$

Revision activity

$$\delta_{c,t} + \delta_{c,t}^{\text{revision}} \leq 1 \quad \forall c \in C, t \in T \tag{32}$$

Start/stop revision of unit

$$\sum_{r \in R_c} \delta_{c,r,0}^{\text{revisions,indic}} = \delta_{c,0}^{\text{revision}} \qquad \forall c \in C \tag{33}$$

$$\sum_{r \in R_c} \delta_{c,r,t}^{\text{revisions,indic}} \geq \delta_{c,t}^{\text{revision}} - \delta_{c,t-1}^{\text{revision}} \quad \forall c \in C, t \in T | 0 < t \leq |T| - \rho_{c,|R_c|}^{\text{duration}} + 1 \tag{34}$$

$$\sum_{r \in R_c} \delta_{c,r,t}^{\text{revisions,indic}} = 0 \qquad \forall c \in C, t \in T || T| - \rho_{c,|R_c|}^{\text{duration}} + 1 < t \tag{35}$$

Minimum revision length of unit

$$\sum_{r \in R_c} \sum_{\substack{\tau=t-\rho_{c,r}^{\text{duration}}+1 \\ \tau \geq 0}}^{t} \delta_{c,r,\tau}^{\text{revisions,indic}} = \delta_{c,t}^{\text{revision}} \quad \forall c \in C, t \in T \tag{36}$$

Yearly revisions per unit

$$\sum_{t \in T} \delta_{c,r,\tau}^{\text{revisions,indic}} = 1 \quad \forall c \in C, r \in R_c \tag{37}$$

Revision order

$$\sum_{t \in T} t\delta_{c,r,t}^{\text{revisions,indic}} + \rho_{c,r}^{\text{duration}} + \rho_c^{\Delta} \leq \sum_{t \in T} t\delta_{c,r+1,t}^{\text{revisions,indic}} \quad \forall c \in C, r \in R_c | r < |R_c| \tag{38}$$

Storage Constraints

Storage fill level definition

$$W_{s,t} = \dot{W}_{s,t}^{\text{load}} \eta_s^{\text{load}} - \dot{W}_{s,t}^{\text{unload}} + W_{s,t-1} \quad \forall s \in S, t \in T | t > 0 \tag{39}$$

$$W_{s,0} = \dot{W}_{s,0}^{\text{load}} \eta_s^{\text{load}} - \dot{W}_{s,0}^{\text{unload}} + W_s^{\text{init}} \quad \forall s \in S \tag{40}$$

$$W_{s,t} = W_s^{\text{init}} \quad \forall s \in S, t \in T | \mod(t+1, \theta) = 0 \tag{41}$$

Maximum storage fill level

$$W_{s,t} \leq w_s^{\max} \quad \forall s \in S, t \in T \tag{42}$$

Minimum storage fill level

$$W_{s,t} \geq w_s^{\min} \quad \forall s \in S, t \in T \tag{43}$$

Maximum storage loading

$$\dot{W}_{s,t}^{\text{load}} \leq \dot{w}_s^{\text{max,load}} \delta_{s,t}^{\text{load}} \quad \forall s \in S, t \in T \tag{44}$$

Maximum storage unloading

$$\dot{W}_{s,t}^{\text{unload}} \leq \dot{w}_s^{\text{max,unload}} \delta_{s,t}^{\text{unload}} \quad \forall s \in S, t \in T \tag{45}$$

Storage activity

$$\delta_{s,t}^{\text{load}} + \delta_{s,t}^{\text{unload}} \leq 1 \quad \forall s \in S, t \in T \tag{46}$$